úterý 28. května 2024 10:15

(1) $f(x,y,z) = x^3 - 2x^2 + y^2 + z^2 - 2xy + xz - yz + 3z \quad , \quad (x,y,z) \in \mathbb{R}^3$.

$\nabla f = (3x^2 - 4x - 2y + z \, , \, 2y - 2x - z \, , \, 2z + x - y + 3)$

$$\nabla f = 0 \Rightarrow \begin{array}{r} 3x^2 - 4x - 2y + z = 0 \\ 2y - 2x - z = 0 \\ 2z + x - y + 3 = 0 \end{array} \quad \begin{array}{l} \rightarrow 0 = 3x^2 - 6x = 3x(x-2) \rightarrow x = 0,2 \\ \rightarrow 0 = 3z + 6 \rightarrow z = -2 \rightarrow 2y = z + 2x = -2 + 2x \end{array} \Bigg\} \rightarrow \begin{array}{l} (0,-1,-2) \\ (2,1,-2) \end{array}$$

$$H_f = \begin{pmatrix} 6x-4 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \qquad H(0,-1,-2) = \begin{pmatrix} -4 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{array}{l} D_1 = -4, D_2 = -12, D_3 = -18 \\ \longrightarrow \text{IND} \rightarrow \boxed{\text{sedlový bod}} \end{array}$$

$$H(2,1,-2) = \begin{pmatrix} 8 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{array}{l} D_1 = 8, D_2 = 12, D_3 = 18 \\ \rightarrow \text{PD} \rightarrow \boxed{\text{lok. minimum}} \end{array}$$

$f(x,0,0) = x^3 - 2x^2 \xrightarrow{x \to \pm\infty} \pm\infty \rightarrow$ globální extrémy neexistují

(3) $f(x,y,z) = x^2 + y^2 + z^2 \quad , \quad M = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 5 \, , \, x - 2y + z = 10\} \leftarrow$ kompaktní $\rightarrow$ existují extrémy

$g(x,y,z) = x^2 + y^2 - 5 \qquad h(x,y,z) = x - 2y + z - 10$

$$\begin{array}{l} \nabla g = (2x, 2y, 0) \\ \nabla h = (1, -2, 1) \end{array} \Bigg\} \rightarrow \text{lin. závislé} \Leftrightarrow x = y = 0 \quad (\text{ve sporu s } x^2 + y^2 = 5)$$

Lagrangeovy multiplikátory $\qquad \nabla f = (2x, 2y, 2z)$

$$\left.\begin{array}{r} 2x = 2\lambda x + \kappa \\ 2y = 2\lambda y - 2\kappa \\ 2z = \kappa \end{array}\right\} \nabla f = \lambda \nabla g + \kappa \nabla h \qquad (*)$$

$$\left.\begin{array}{r} x^2 + y^2 = 5 \\ x - 2y + z = 10 \end{array}\right\} g = h = 0 \qquad (**)$$

$x^2 = 4y^2 + 40y + 100$

$\to y^2 + 8y + 19 = 0$

$64 - 76 < 0$

$(*) \to (\lambda - 1)(2x + y) = 0$

$\lambda = 1 \to \kappa = 0 \to z = 0 \to x - 2y = 10$ (nemá řešení)

$y = -2x \to y^2 = 4x^2 \to 5x^2 = 5 \to x = \pm 1 \to y = \mp 2 \to z = 5, 15$

$f(1, -2, 5) < f(-1, 2, 15)$

(minimum) (maximum)